Mesoscopic theory of optical wavefront shaping to focus deep inside opaque media

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Abstract. We describe the theory of optical wavefront shaping, where waves are focused to a predefined focal point deep inside a disordered three-dimensional (3D) opaque medium (*e.g.*, tissue, paint, foam) by shaping the wavefield sent by N external array elements. Our work is motivated by recent studies where wavefront shaping (controlled interference of scattered waves) coexists with classical diffusion for which wave interference appears to be irrelevant. We derive the energy density both near the focus and anywhere in the medium, averaged over realizations after optimization. We find that the average energy density including focusing is described by the C_1, C_2, C_3 and C_0 intensity correlations known from mesoscopic transport theory. Remarkably, the background energy density obeys a diffusion equation wherein C_2-correlations create an energy source inside. Thus, a classical property (diffusion) coexists with interference. We discuss several energy density profiles proposed in literature, that are associated with optimized transmission by a slab using wavefront shaping. Our results are relevant for applications where the internal energy density in opaque media is crucial, such as in white-light illumination, projection optics, semiconductor metrology, (bio)sensing, and photovoltaics.

We describe the theory of focusing waves to a predefined focal point deep inside disordered 3D opaque media by shaping the field of N external sources, also known as optical wavefront shaping [1]. Our study is motivated by experiments where wavefront shaping, mediated by controlled interference of scattered waves, appears to surprisingly coexist with a classical-diffusion picture where interference seems to be absent in 3D, 2D, and 1D [2-4].

We derive the energy density of the wave field both near the focal point and anywhere inside the medium, averaged over realizations after optimizing the focus. We thus conceive of a point source at the focus deep inside, that emits waves to an external detector array that - by time reversal - becomes a source array that emits the desired shaped fields like a spatial modulator, see Figure 1(a) [5]. The theory shows that the focus is described by constructive interference induced by C_1 correlations, known from Gaussian speckle in mesoscopic

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transport theory [6], whereas the averaged energy density of the background is described by C_2 and C_0 correlations.

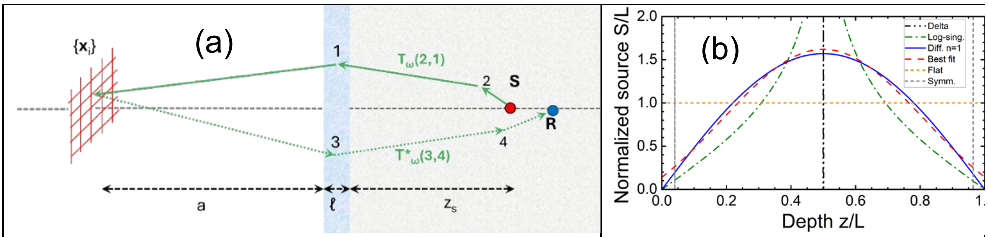

Fig. 1 (a) Schematic of wavefront shaping to focus to a point S in a disordered 3D medium with a skin layer at the surface (δz ≈ℓ). A wave packet from a virtual source at S goes to the detector array with N detectors far away (left). Optimal shaping is achieved by time-reversing the wave packet that refocuses on a diffraction-limited spot R≈S with a background energy calculated here [5]. **(b)** 6 different source distributions from literature: Delta function, log-singular, diffusion n = 1 mode, best fit, flat, symmetric.

We draw the remarkable conclusion that by averaging over optimized wavefronts of the array (that each optimize the focus for different realizations of the disorder) an average background energy density is obtained that obeys a diffusion equation in which C_2-correlations create an energy source around the focus. We systematically derive the peak and background energy density distributions for C_0, C_1, C_2-correlations for both an half-space and finite slabs. We find the relative enhancement of the energy density at the focus to be proportional to the angular opening of the array (SLM, DMD), which *de facto* replaces the role of the inverse dimensionless conductance 1/g as the universal parameter in mesoscopic transmission statistics, and is proportional to the number of segments in experiments.

We exploit the concept of a "source distribution" in the medium for the "*optimized background energy*" to discuss 6 different profiles proposed in literature, that are associated with optimal transmission by a slab using wavefront shaping. We find that the resulting energy densities are remarkably similar, with the best matching similar to the dominant diffusion mode, agreeing with a previous conjecture [2]. The simultaneous coexistence of classical diffusion and interference is crucial.. Our approach avoids random-matrix theory that is restricted to waveguide geometries, confirms the importance of non-Gaussian correlations in the wavefront shaping for focusing deep inside 3D random media, and highlights the notion of an energy source created in this process.

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