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Localization and increased damping in irregular acoustic cavities

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Abstract

The present study is concerned with the properties of 2D shallow cavities having an irregular boundary. The eigenmodes are calculated numerically on various examples, and it is shown first that, whatever the shape and characteristic sizes of the boundary, irregularity always induces an increase of localized eigenmodes and a global decrease of the existence surface of the eigenmodes. Besides, irregular cavities are shown to exhibit specific damping properties. As expected, the increased damping, compared to a regular cavity, is related first to the larger perimeter to surface ratio. But more interestingly, there is a specific enhancement of the dissipation for those modes that are localized near the boundary, modes which are favored by the geometric irregularity.

1. Introduction

It is classical in room acoustics to cover walls with an absorbing material to damp acoustic waves. One may even use ‘‘rough’’ absorbers, such as curtains, foam wedges or profiled diffusers [1,2], to increase the absorption efficiency. A first guess would suggest that the damping is proportional to the ‘‘amount’’ of absorbing material put in the room but this is not true: if the geometry is irregular, the increase in damping is ‘‘more than proportional’’. Recently, these ideas have been used to design a new type of efficient road noise barrier (the Fractal Wall™, product of Colas Inc., French patent No. 0203404). In this application the effects of the irregular geometry and bulk absorption in the material are combined. It should be noted that the role of specific geometry designs to increase bulk absorption has received recent interest [3–5].

Here we consider the surface absorption on an irregular wall. More precisely, we show that there exists a specific physical mechanism, namely localization, which creates an enhancement of damping, so that the absorption increases more rapidly than the surface of absorbing material. For localized modes, the so-called existence volume may be only a small fraction of the total resonator volume and this fraction decreases when the irregularity is more pronounced. These localized modes are found to be confined near the boundary where the dissipation occurs. So, for these modes, localization directly contributes to the enhanced damping power

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of such irregular cavities. These effects were first found in studies of prefractal acoustic cavities [6–8]. But if one searches for more realistic damping structures in, e.g. architectural or urban acoustics, the use of fractal or prefractal structures may be practically difficult.

This raises the question of whether the increased damping is related specifically to fractality, or more simply, and more generally to geometric irregularity itself. Recently, the damping in non-fractal rough drums with Dirichlet boundary condition has been studied [9]. It was found that, in specific cases, rough Dirichlet boundaries may increase the damping. In the following we show that the three properties displayed by prefractal acoustic cavities—enhanced density of states, existence of localized modes (see also Ref. [10]), and increased damping—are general properties of any irregular structure and increase with geometry irregularity. Notably, the appearance of localization linked to irregularity and the corresponding damping enhancement are found to be the properties of any irregular cavity *whatever its shape and characteristic sizes*.

2. Localization in irregular cavities

In this section we show that, in a 2D shallow cavity with irregular rigid boundary Γ, there always exist a significant number of modes which are confined close to the irregular region of the cavity. As model cavities we consider a square cavity of side a where the whole or part of the upper side is replaced by irregularly shaped walls, as shown in Fig. 1. In order to allow comparisons between different geometries, the surface area is kept constant, equal to $S = a^2$, independently of the cavity shape.

Assuming an adiabatic linear lossless medium and weak losses at the wall—the surface admittance $\varepsilon(\omega)$ of the wall is supposed to be very small—the amplitude distribution of the eigenmodes is well approximated by the zero-loss eigenmodes, solutions of the eigenvalue problem

$$\nabla^2\psi_n = -k_n^2\psi_n, \tag{1}$$

with the boundary condition at the walls

$$\mathbf{n}\cdot\boldsymbol{\nabla}\psi_n = 0, \quad \mathbf{n} \text{ the normal vector to the boundary.} \tag{2}$$

The eigenvalue k_n of the above problem is taken as approximated eigenfrequency. The case of arbitrary losses is discussed later.

The localization or confinement of the eigenmode ψ_n is characterized by its "existence surface" [11,6]:

$$S_n = \frac{1}{\int_{\mathrm{D}} |\psi_n|^4 \, \mathrm{d}S}, \tag{3}$$

where ψ_n is normalized by

$$\int_{\mathrm{D}} |\psi_n|^2 \, \mathrm{d}S = 1. \tag{4}$$

According to this definition, a mode will be considered as being *localized* if its existence surface S_n is significantly smaller than the surface area $S = a^2$ of the cavity. In a square (or rectangular) cavity, with classical cosine eigenmodes ψ_{mn} $(m, n \in \mathbb{N})$, the relative existence surface S_{mn}/S is 1 for $m = n = 0$, $\frac{2}{3}$ for m or $n = 0$, and $\frac{4}{9}$ for m and $n>0$. There are no localized modes in such a particular geometry.

2.1. Localization increases with irregularity

The numerical results described in the following have been obtained from numerical computations of the eigenvalues k_n and eigenfunctions ψ_n using a P2 finite elements scheme and the solving libraries from FEMLAB®. A few examples are shown in Fig. 1. For the six different irregular cavities, one particular localized mode has been pictured. For all these cases, one observes the strong confinement of the amplitude distribution in the vicinity of the irregular wall. This generalizes what was shown on prefractal cavities: fractality is not a prerequisite for localization. Any irregular cavity will exhibit localized modes, whatever the shape and characteristic sizes of its boundary.

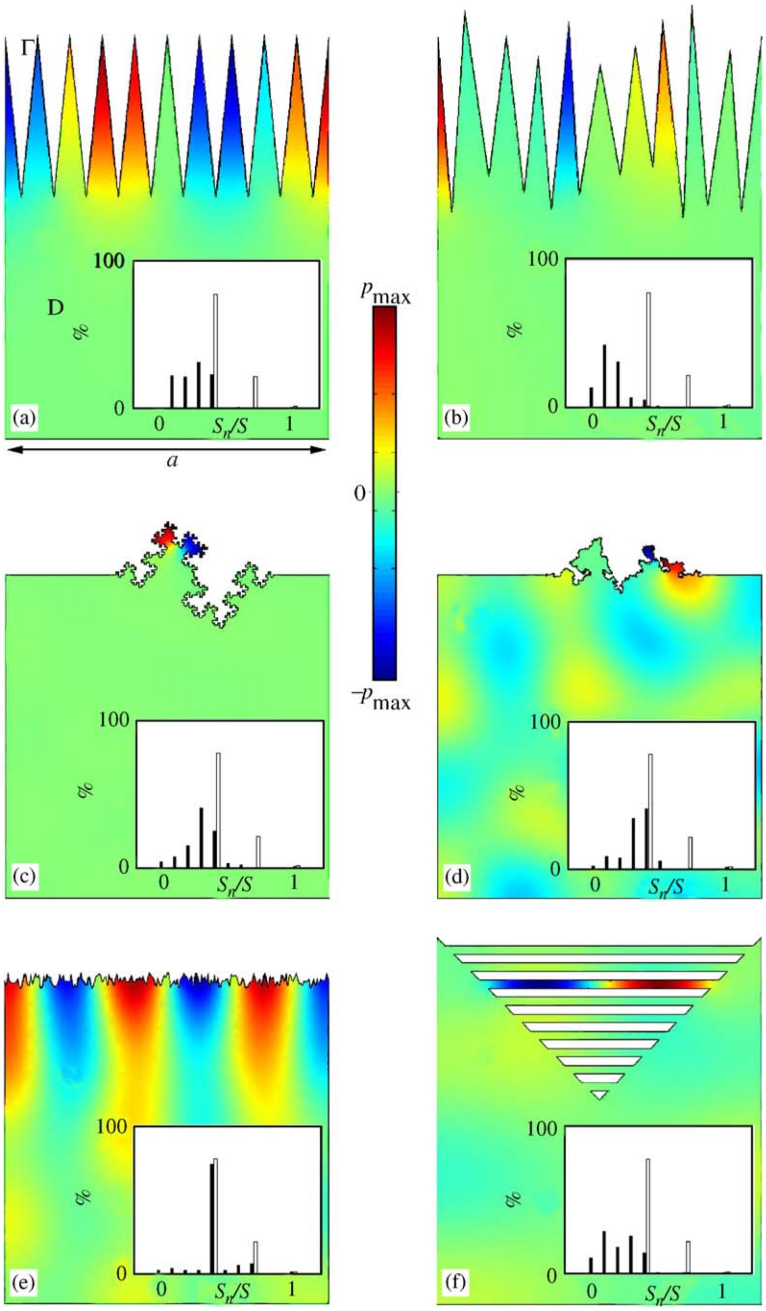


Fig. 1. Examples of the amplitude distribution of localized modes in different irregular shallow cavities. One observes that, whatever the type of irregular geometry, there always exist localized modes, i.e. modes which exhibit a very small amplitude in a major fraction of the cavity space. The insets give for each geometry, the histogram of the relative existence surface of the eigenmodes in the frequency range $k_n a/\pi \in [0, 10]$, compared to the results for a square cavity indicated by the empty bars (Color online).

The insets in Fig. 1 give for each cavity the histogram of the relative existence surface S_n/S of the eigenmodes in the frequency range $k_n a/\pi \in [0, 10]$, compared to the results for a square cavity. The effect of the irregularity clearly appears on these plots: in each case, apart from the fundamental mode whose relative existence surface is equal to 1 independently of the geometry, almost all of the modes have a relative existence surface smaller than $\frac{4}{9}$, the minimum value in a square cavity.

Moreover, as it was shown for prefractal cavities, this tendency to localization is increased by the irregularity of the boundary. To show this effect one considers, as an example, the geometry shown in Fig. 1a, where the upper side of the initial square cavity has been replaced by N identical triangular wedges of height $a/2$. The surface area of the cavity, $S = a^2$, is independent of N and the perimeter length is a linear function of $\sqrt{1+N^2}$ and thus increases as N. Fig. 2 shows the relative existence surface S_n/S of the first eigenmodes ($k_n a/\pi \in [0, 10]$) of the cavity with 5, 10 and 20 wedges, respectively, as a function of their eigenfrequencies. One observes a general decrease of the mean existence surface with increased irregularity. (Note that in this particular example, it is the perimeter to surface ratio that is chosen to characterize the irregularity of the boundary. For non-fractal geometries, and our purpose in this paper is precisely to consider non-fractal geometries, it is a simple way to measure the irregularity, although it is not fully satisfactory. In the following paragraph, a cavity with the same surface area and perimeter to surface ratio that the one described above, but "disordered" and in that sense more irregular, is considered to complete our study.)

Two comments can be formulated about these results. First, the total number of modes in the chosen frequency interval increases with the perimeter to surface ratio. This reflects the classical correction to the Weyl leading term in the density of modes [12]. Second, the modes can be split into two families. A first group, the non-localized modes, is composed of the modes with typically, $S_n/S > 0.2$. The number of these modes is globally independent of the number of wedges. Then, there is a second group, composed of an increasing number of very localized modes with $S_n/S < 0.2$. These modes are confined near the irregular wall, in the "sub-cavities" formed by the wedges (Fig. 1c), and their eigenfrequencies are grouped near frequencies

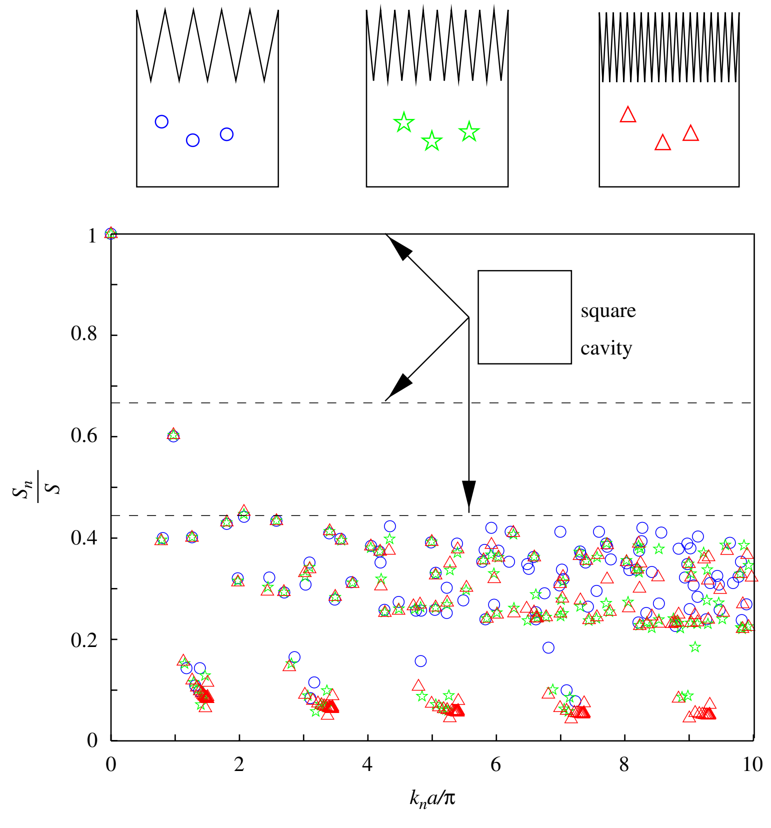


Fig. 2. Relative existence surface of the first eigenmodes ($k_n a/\pi \in [0, 10]$) of the cavity of type (a) (see Fig. 1) with (○) 5; (✳) 10 and (△) 20 wedges, respectively. The dashed lines indicate the possible values for a square cavity. One observes that the more irregular the boundary the more localized the eigenmodes (Color online).

$ka/\pi \sim 1, 3, 5, 7, \ldots$. If one considers the sub-cavities as one-dimensional resonators with length $a/2$, these frequencies correspond to $\lambda/4, 3\lambda/4, \ldots$ resonances.

2.2. Effect of randomization of the geometry

In the preceeding example—N identical wedges—the geometric irregularity of the cavity is, in fact, quite "regular". This specific shape limits the confinement of the amplitude distribution of the modes, because of the degeneracy of the eigenmodes appearing in the irregular region.

Thus one expects that breaking this "regularity"—the periodicity of the upper wall shape—, without changing the perimeter to surface ratio, will increase the irregularity, and therefore the localization of the modes. To this end, one may modify the geometry of the cavity shown in Fig. 1a by randomly varying, here up to 20% of their initial value, the base and height of each wedge, as well as the abscissa of each vertex (Fig. 1b). In this particular numerical calculation, we have ensured that the surface area and perimeter length of the cavity differ by less than 0.01% from their initial value.

Results are shown in Fig. 3, where the relative existence surfaces of the eigenmodes in a cavity with 10 identical wedges are compared to those in a cavity with 10 "disordered" wedges. The net effect of breaking the periodicity of the initial shape is to increase localization. Note that the relative existence surface of some modes is as low as 1%.

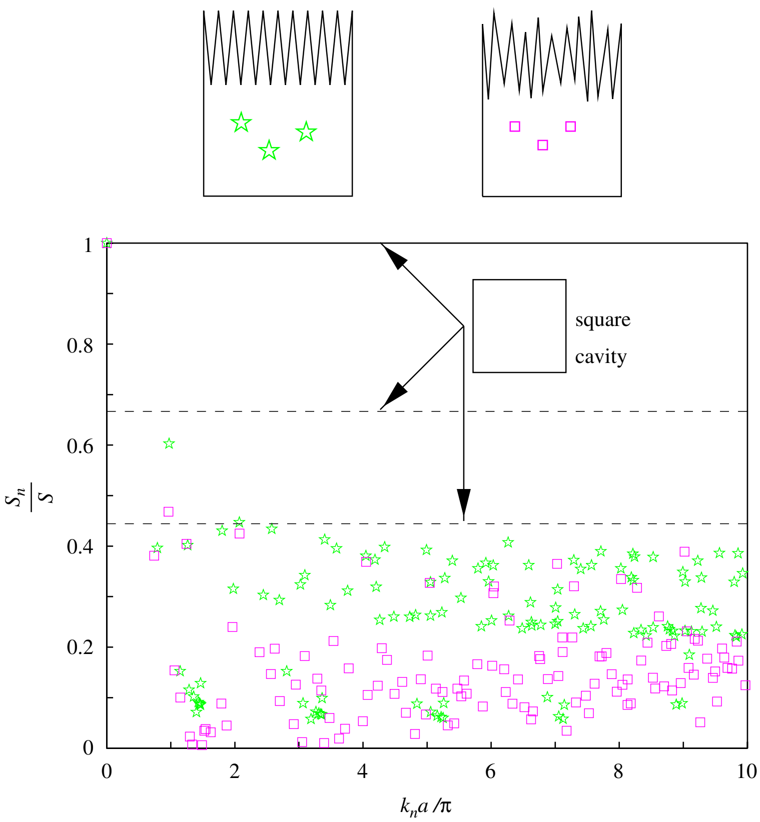


Fig. 3. Comparison between "regular" and randomized geometric irregularities, here a random set of wedges. The figure gives the relative existence surface of the first eigenmodes ($k_n a/\pi \in [0, 10]$) of two cavities of identical surface area and perimeter length: (✳) cavity of type (a) (see Fig. 1) with 10 identical wedges, (□) cavity of type (b) with 10 "disordered" wedges. The dashed lines indicate the possible values for a square cavity (Color online).

2.3. The mechanism of localization

In the above results, one can note that any irregularity with a given size generates one or several localized modes with a wavelength commensurate with that size. An example is the mode shown in Fig. 1c. But more subtle cases exist, such as the mode shown in Fig. 1d which do not pertain to a visible geometric feature, or Fig. 1e.

The existence of localized modes raises the question of the physical mechanism responsible for this phenomenon. First note that, in contrast with fractal drums in which some of the modes are confined by the Dirichlet boundary condition which forbids propagation in narrow regions, here there is no such "confining force". Here the localization effects result from spatial coherence effects, or, in other words, from the fact that diffractive effects create constructive interferences only in a small fraction of the resonator volume. The interferences are essentially destructive elsewhere [13].

Then, in contrast with the first mentioned mechanism that results in a strong localization of the wave and an exponential decay of its amplitude, here the localization is weak, characterized by a much slower decrease in space. Our case is displayed in Fig. 4. The top figure illustrates the spatial dependance of a localized state in a resonator. The bottom figure shows the same state in "open space" with non-reflecting boundary conditions on the three sides of the square replacing the Neumann conditions (perfectly matched layers [14] were implemented in FEMLAB for the numerical computations). Fig. 4 shows a rapid decrease of the amplitude in the vicinity of the zone of confinement and a slower decrease (much slower than exponential) at larger distance from the zone of confinement. This slow decrease is characteristic of this weak localization.

2.4. Mode repulsion

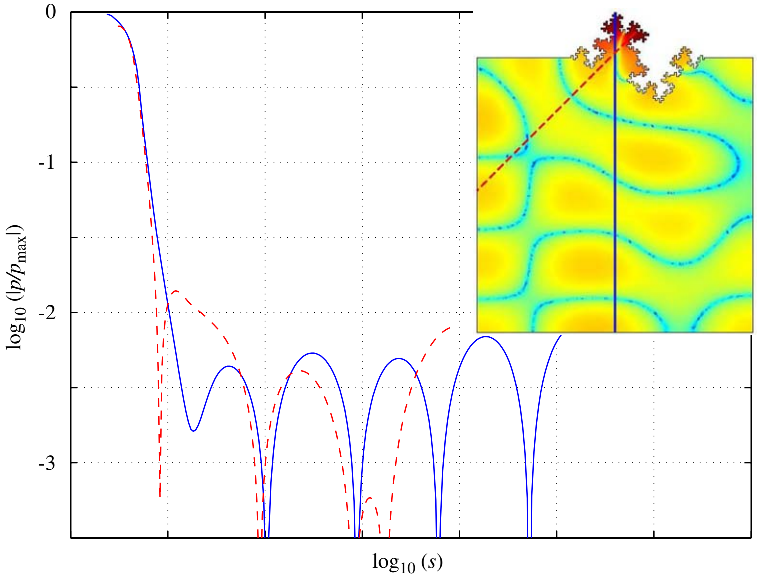
To be complete, one should mention that a closer look at the modes of the above cavity (Fig. 1a), but also of other geometries of cavity based on a square, reveals that some of the non-localized states exhibit very weak amplitudes near the irregular wall. Such modes are shown in Fig. 5. These modes are obviously very close, by either their eigenvalue ($ka/\pi \sim m$, $m \in \mathbb{N}$) or their amplitude distribution in the x direction (parallel to the bottom wall), to the "unperturbed" modes $\psi_{mn} = \sqrt{(2-\delta_{m0})(2-\delta_{n0})}a^{-1}\cos(m\pi x/a)\cos(n\pi y/a)$ of a square cavity, with $n = 0$. But for those modes, the amplitude distribution in the y direction is strongly modified, with a rapid decrease towards the irregular part of the boundary. Thus, although the irregularity has been found to generally confine many modes in its vicinity, it appears that for a few modes the effect is in fact a "repulsion" effect. A similar phenomena has been found and analyzed by Izrailev et al. in quasi-1D waveguides with a rough surface and Dirichlet boundary conditions [15]. One can understand qualitatively the existence of these modes in a perturbative approach. If, instead of a geometry based on a square, one would consider a long rectangle, with only one of the shorter sides made irregular, the non-localized modes would be essentially those of the rectangle. But as those modes cannot satisfy the Neumann boundary condition of the true rectangle they have to be very weak in the region of the irregular wall. This argument is sustained by the fact that, if one compute the eigenmodes of an irregular cavity based not on a square but on a trapezoid shape, this "repulsion effect" tends to vanish. In summary, the repulsion effect is at least partially a consequence of our choice to study square-based cavities.

2.4.1. Propagation in irregular waveguides

In the frequency domain, the sound field—the acoustic pressure p—for propagation in a cylindrical waveguide can be expressed as the infinite series

$$p(s, \mathbf{w}) = \sum_{n\in\mathbb{N}} \psi_n(\mathbf{w})(A_n^- e^{-j\gamma_n s} + A_n^+ e^{j\gamma_n s}), \tag{5}$$

where s and $\mathbf{w}$ are the longitudinal and transverse coordinate, respectively, $\psi_n(\mathbf{w})$ are the transverse modes in the waveguide, that is, the solutions of the transverse Laplacian eigenproblem $\nabla_\perp^2 \psi_n = -k_n^2 \psi_n$, and γ_n are the longitudinal wavenumbers: $\gamma_n^2 = k^2 - k_n^2$, $k = \omega/c_0$ being the wavenumber. If now one supposes that the cross-section of the waveguide is irregular, so that some of the transverse modes are localized in a restricted region



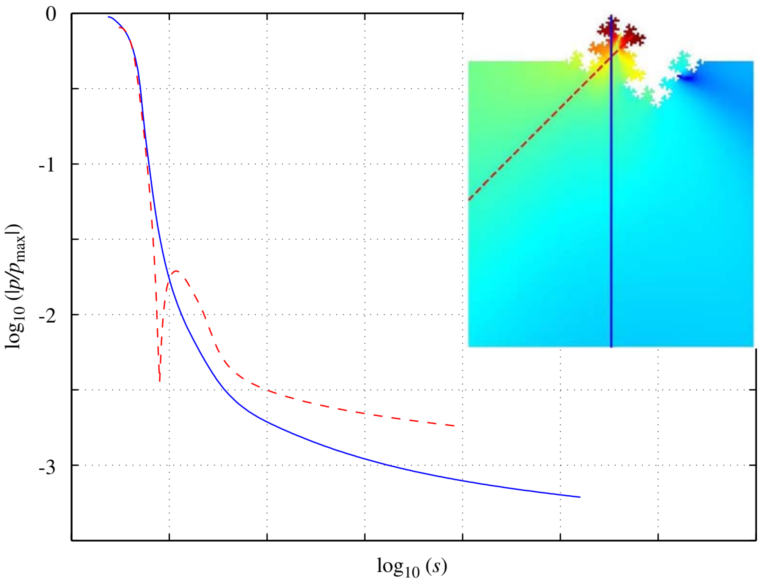


Fig. 4. Decrease of the amplitude of a localized mode in two cases: top: the mode in a cavity with a prefractal defect (Fig. 1c). Bottom: the same mode in an open space limited by the same prefractal frontier, and appearing with almost the same eigenfrequency. The red dashed curved gives the amplitude along the corresponding red line on inset pictures, and similarly for the blue solid curves. s is the arc length along these lines (Color online).

of the cross-section, such a waveguide may be used to propagate simultaneously different acoustic signals that would be spatially separated by 2D localization.

3. Energy dissipation

We now discuss the impact of the irregularity on the damping properties of the cavities. The medium filling the cavity is supposed to be lossless and we consider the cases of weak and arbitrary losses occurring at the walls, having a finite admittance $\varepsilon(\omega) \in \mathbb{C}$.

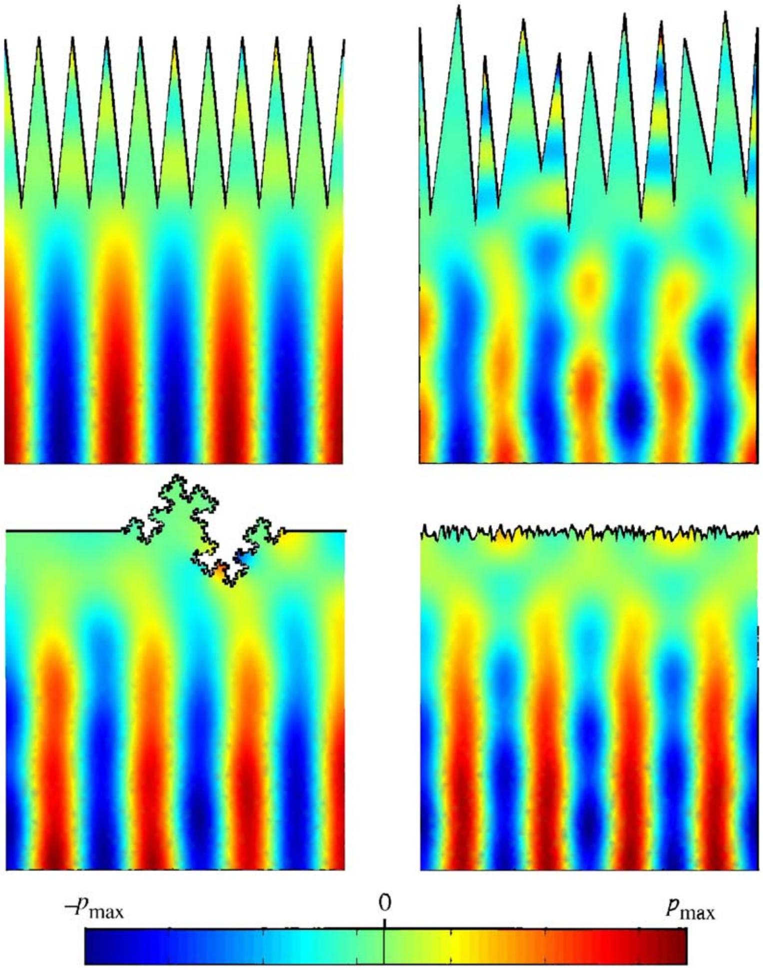


Fig. 5. Illustration of the repulsive effect of the irregularity (Color online).

3.1. Weak losses

If $p_n = P_n S^{1/2}\psi_n$ is the pressure field corresponding to the normalized eigenmode ψ_n, the associated dissipated energy W_n, that is the total outflow through the cavity walls, is

$$W_n = \int_\Gamma \frac{|p_n|^2}{2\rho_0 c_0}\,\mathrm{Re}(\varepsilon)\,\mathrm{d}L \tag{6}$$

or

$$W_n = \frac{P_n^2 S}{2\rho_0 c_0}\,\mathrm{Re}(\varepsilon)\int_\Gamma |\psi_n|^2\,\mathrm{d}L. \tag{7}$$

Then, the energy dissipation is a direct function of the amplitude distribution of the mode in the cavity by the term

$$w_n = \int_\Gamma |\psi_n|^2\,\mathrm{d}L. \tag{8}$$

The role of localization is schematically illustrated in Fig. 6. Consider a non-localized mode (or the fundamental mode $\psi_0 = 1/\sqrt{S}$) of a cavity with a simple irregularity as shown on the left of the figure. Its normalized amplitude $|\psi_0|^2$ is of order $1/S$ and the associated loss term w_0 is thus L_p/S, where L_p is the

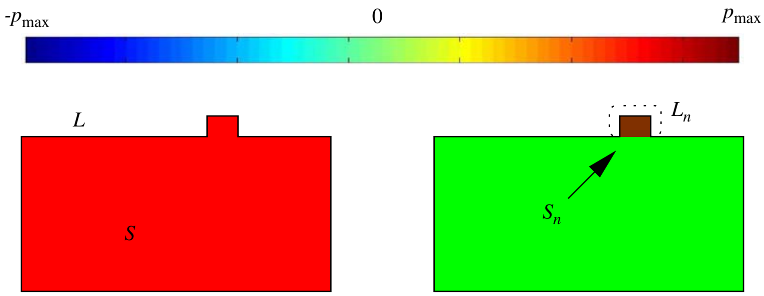


Fig. 6. Simplified explanatory scheme for the different dissipation properties of a localized and a non-localized mode in an irregular cavity (Color online).

perimeter length of the cavity. Therefore, for non-localized modes, the energy dissipation is roughly proportional to the perimeter length, always an increasing function of irregularity. (For a square cavity with eigenmodes ψ_{mn}, w^s_{mn} is equal to $4/a$ for $m = n = 0$, $6/a$ for m or $n = 0$, and $8/a$ for m and $n > 0$.)

Consider now some particular mode ψ_n localized in the upper right defect. In a very crude approximation, $|\psi_n|^2$ is of order of $1/S_n$ so that the loss term w_n for this mode is L_{pn}/S_n, where L_{pn} is the perimeter of the zone where the mode exists. This tells us that the more localized and the longer the perimeter of the localization region, the more damped the mode will be. This would be particularly true if the perimeter of the localization zone would be large as in the case of a prefractal curve of higher generation.

Again, consider the cavity of Fig. 1a. The energy dissipation of the modes computed through Eq. (8) is given in Fig. 7a and b. To clearly point out the effect of the localization on the energy dissipation, the loss term w_n is plotted twice: on plot (a) it is normalized by the losses w^s_{00} of the fundamental mode in a square cavity of surface S, in order to show the global effect of the increase of the irregularity, and on plot (b) it is normalized by the losses $w_0 = L/S$ of the fundamental mode in the cavity that is considered. With this normalization, the energy dissipation of the fundamental mode is 1 for any cavity. Therefore, what increases proportionally to the perimeter in the losses is hidden, and the sole effect of localization is shown.

For almost all of the modes the dissipation is increased as compared with a square cavity. This plot allows us to distinguish the two families mentioned earlier (cf. Section 2.1). First, the non-localized modes, appearing now to correspond to the weakly dissipative modes (with typically, $w_n/w_0 < 4$, see plot (b)). Second, the strongly localized modes, corresponding to the strongly dissipating modes. Consider now how the energy dissipation of the modes varies with the geometric irregularity of the boundary. If the energy dissipation of the non-localized modes increases in a manner that is roughly proportional to the perimeter length, as expected, the energy dissipation of the more localized modes increases more rapidly, due to the combined effects of the increase of the perimeter length and of localization.

More generally, the dependance of the energy dissipation with the localization is shown in Fig. 7c and d. This plot of the energy dissipation as a function of the relative existence surface reveals a global trend characterized by an increase of the damping with irregularity and localization. This dependence of the energy dissipation of a mode with its degree of localization results from the above discussion. Again as mentioned, breaking the periodicity of the upper wall shape by randomly varying the dimensions of the wedges increases significantly the localization, without changing of the perimeter to surface ratio. Consequently, as localization and damping are strongly related, the energy dissipation also increases when "disordering" the wedges.

Note that the modes that are "repulsed" by the irregular boundary clearly appear when plotting the losses (Fig. 7a and b; as precised earlier, these modes appear at frequencies $ka/\pi \sim n$, $n \in \mathbb{N}^*$): since these modes "touch" only the 3 regular walls of the cavity and consequently a small part of the length of the whole boundary, the corresponding losses are relatively small.

3.2. Arbitrary losses

For reasons of simplicity, the properties of localization and the resulting enhancement of the energy dissipation have been shown under the assumption of weak losses at the wall, i.e. when the surface admittance

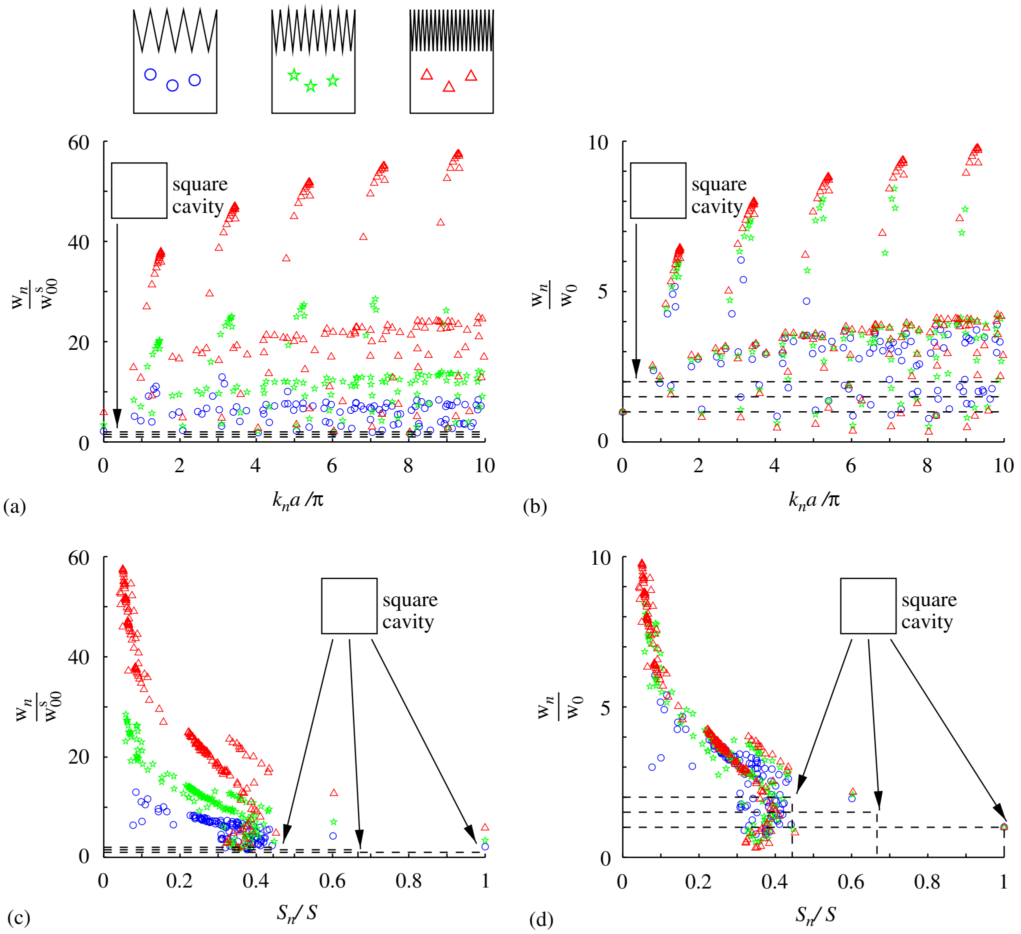


Fig. 7. (a) Energy dissipation, as measured by w_n/w_{00}^s, for the first eigenmodes ($k_n a/\pi \in [0, 10]$) of the cavity of type (a) (see Fig. 1) with (○) 5, (✳) 10 and (△) 20 wedges, respectively. (b) same results, with w_n normalized by $w_0 = L/S$ for each cavity, in order to emphasize the specific role of the localization. (c–d) Energy dissipation vs. the relative existence surface. The dashed lines indicate the possible values for a square cavity (Color online).

$\varepsilon(\omega)$ is very small. Various examples of cavities with functions ε that are not small have been also studied. In view of the results—an example is given below—one can reasonably conjecture that, qualitatively, these properties can be generalized to arbitrary values of the surface admittance ε.

Consider, as an example, a 2D square cavity, of which three sides are rigid and perfectly reflecting while the fourth (the upper side) is lined with a layer of rigid porous material. Assuming a locally reacting material, the reduced surface admittance $\varepsilon(\omega)$ is thus taken as $\rho_0 c_0/z(\omega)$, where $z(\omega)$ is the surface impedance at normal incidence of the porous layer:

$$z(\omega) = \frac{j}{\phi}(\rho K)^{1/2} \cot\left(\omega\left(\frac{\rho}{K}\right)^{1/2} d\right), \tag{9}$$

here d is the layer thickness of the porous layer, ϕ the porosity, $\rho(\omega)$ the effective density and $K(\omega)$ the effective bulk modulus of air in the material (on the dependance on ρ and K with the characteristic parameters of the porous material, see, e.g., Ref. [16]). Besides, as this admittance is frequency dependent, we replace this function by a piecewise constant approximation and solve the eigenvalue problem in each of the defined frequency intervals. In the following results, the cavity side is $a = 0.2$ m and the porous materials is characterized with the following parameters: porosity $\phi = 0.98$, tortuosity $\alpha_\infty = 1.2$, flow resistivity

$\sigma = 40000\,\mathrm{N\,m^{-4}\,s}$, characteristic dimensions $\Lambda = 2 \times 10^{-4}$ m and $\Lambda' = 4 \times 10^{-4}$ m, thickness $d = 0.1$ m. With these parameters, $|\varepsilon|$ varies between 0 and 0.9 in the frequency range that we consider and cases of strong absorption are then taken into account (note the ε is a normalized admittance).

The plane upper wall that is lined is then replaced by an irregularly shaped wall, and one supposes that this new boundary has the same surface admittance as the porous layer preceedingly mentioned.

The comparison between these two cavities is shown in Fig. 8. As found in the weak losses case, the irregular geometry of the boundary induces a global effect of confinement of the modes, as shown on the first plot giving the relative existence surfaces. Due to this increase of the amplitude in the irregular and lossy region, the damping of the modes is also enhanced (Fig. 8b). For these results, the integral path in Eq. (8) has been restricted to the irregular boundary.

4. Conclusion

In summary, it has been shown that whatever its specific shape, the irregular morphology of a 2D shallow acoustic cavity contributes to enhance its dissipative power. This has been shown for several different types of geometry so that the conclusion must hold quite generally. A first effect is simply due to the enhanced surface of interaction between the acoustic modes and the cavity walls. But more interestingly and quite generally, there appear a number of modes localized near the irregular walls. The damping of these localized modes is specifically increased. The general conclusion is simple and applicable: geometric irregularity increases the effective damping of acoustic cavities.

In this paper, we have been concerned by shallow cavities and the question remains open for 3D cavities. The 3D problem can be splitted in two different questions. First, it is well known that localization effects may be very different in 2D and 3D and this question should be studied in the future for acoustic cavities. But secondly, the relations between localization and damping derived in this work holds for any space dimensionality.

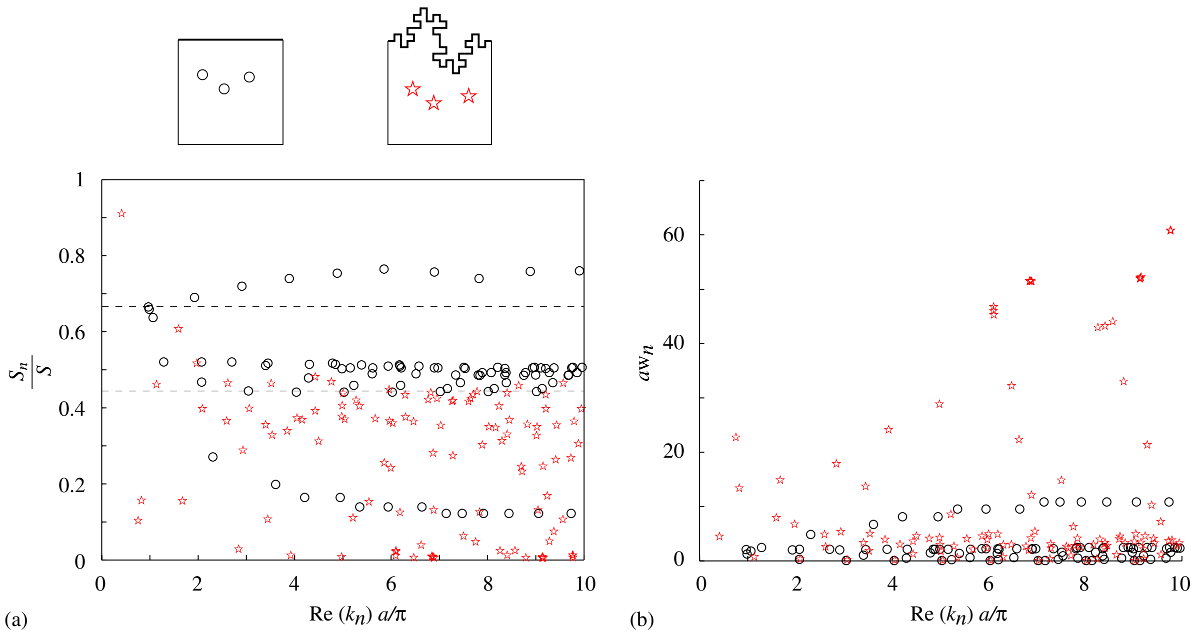


Fig. 8. (a) Relative existence surface and (b) energy dissipation, as measured by w_n (normalized by $1/a$), for the first eigenmodes ($\mathrm{Re}(k_n)a/\pi \in [0, 10]$) of the cavities shown in insets, of which the upper boundary admittance is lined with a porous layer. The dashed lines on plot (a) indicate the possible values for a lossless square cavity (Color online).

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