Investigation of surface acoustic wave propagation on a sphere using laser ultrasonics

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Using the noncontact laser ultrasonic technique, we investigate the propagation of surface acoustic waves on a sphere (SSAW). A finite thermoelastic line source gives rise to combined focusing and reversal effects. As for propagation on a cylinder surface, the reversal of the SSAW pulse is explained by the dispersion of the high frequency components of the laser-generated acoustic pulse. The focusing effect, due to the curvature of the surface, depends on the angular aperture of the source. From diffraction theory, optimal conditions for diffraction free propagation are derived, both in harmonic and pulse regimes. On a steel sphere of radius 25 mm, Rayleigh waves excited by a 20 ns YAG laser pulse focused onto a 3 mm line propagate with a nearly constant amplitude. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1791331]

Surface acoustic waves (SAW) propagating on a sphere have characteristics different from those of Rayleigh waves propagating on a plane. They have been studied by geophysicists[1,2] and also discussed more generally by Viktorov.[3] The excitation and radiation of SAW on a sphere immersed in water have also been investigated as an acoustic scattering problem.[4] De Billy shows the role of SAW in the sound propagation in a chain of spherical beads.[5] First experiments where the waves were generated by a pulsed YAG-laser and detected by an optical heterodyne probe demonstrated the advantages of laser-based ultrasonic techniques.[6,7] Recently, Ishikawa *et al.* excited divergent, focused, and collimated spherical SAW by an interdigital transducer.[8]

In a previous work,[9] we show that a short SAW pulse was reversed during its propagation on a duraluminum or a steel cylinder. This phenomena was explained by the dispersion effect on the large ka components (k: SAW wave number, a: cylinder radius). In this letter, using the noncontact laser ultrasonic technique, we investigate the combined focusing and reversal effects for spherical surface acoustic waves (SSAW).

As for a cylinder, SSAW fall in two categories: the Rayleigh and the so-called whispering gallery waves, related to bulk acoustic waves. The dispersion equation providing the angular frequency ω versus the wave number k is given in Ref. 2. An integer value n of ka corresponds to a resonance frequency ω_n. In our experiments only the Rayleigh mode, giving rise to large surface displacements, is efficiently coupled to the pulsed laser source and to the interferometric laser probe. Figure 1 shows, for a steel sphere of radius a, the Rayleigh wave phase velocity V versus the product ka. The dispersion effect is mostly noticeable for low frequencies ($ka<10$) such that the wavelength λ is larger than $a/2$. As ka tends to infinity (large frequencies), the velocity tends to the value $V_R=2960$ m/s corresponding to the speed of the Rayleigh wave in a thick steel plate. Since this behavior is similar to that of Rayleigh waves propagating on a cylinder, a reversal of the mechanical displacement is expected for a given longitude angle ϕ.[9]

Experiments were carried out on a steel sphere of diameter 50 mm. Surface waves were generated by a Q-switched Nd:YAG laser providing pulses having a duration $\Delta=20$ ns and a 3 mJ energy. A beam expander and a cylindrical lens were used to focus the beam onto a line along the meridian $\phi=0°$ (Fig. 2). The spatial energy distribution, plotted by scanning a narrow slit, was closed to a Gaussian. The full length $2b$ of the thermoelastic source was defined at $1/e$ of the maximum value. The SSAWs propagate along an equator of the sphere perpendicular to the source. The mechanical displacement normal to the surface was measured by a heterodyne interferometer equipped with a 100 mW continuous laser emitting at 532 nm.[10] The calibration factor of the probe (130 mV/nm) was constant over the detection bandwidth (50 kHz–20 MHz). The first arrival of the Rayleigh wave was recorded according to the angle ϕ between the source and the detection point.

Figure 3 shows a series of wave forms from $\phi=20°$ to $\phi=180°$ (step 5°) launches by a line source of length $2b=12$ mm. In the vicinity of $\phi=90°$ two phenomena are observed: the displacement is reversed and the amplitude of the SAW pulse undergoes a maximum. As for a cylinder, the first effect can be explained by the dispersion of the high frequency components (Eq. 8 of Ref. 9). Fitting the tail of the dispersion curve in Fig. 1 ($ka>100$) with a function

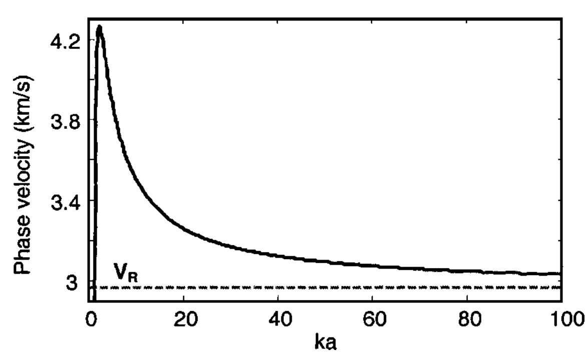

FIG. 1. Rayleigh wave phase velocity vs ka for a steel sphere of radius a.

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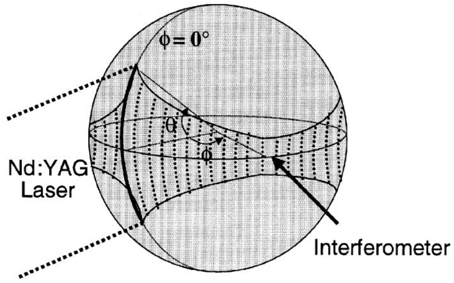

FIG. 2. Schematic of Rayleigh wave generation, propagation, and detection on a sphere. The waves are generated by a pulsed YAG laser focused onto a line of length 2*b*. They are detected by an optical heterodyne interferometer.

$$V = V_R\left(1 + \frac{\varepsilon}{ka}\right) \tag{1}$$

leads for the parameter ε to a value 2.22 which gives a reversal angle $\phi_r=\pi/\varepsilon=81°$ closed to the experimental figure (83°). The second effect is due to the curvature of the surface: the Rayleigh wave converges toward the point on the equator located at $\phi=90°$. Figure 4 shows the profiles of the SAW beam measured by scanning the optical probe beam perpendicularly to the equator at $\phi=20°$ and 90°. Compared to the source length, the beam compression factor is equal to 3.5. If this focusing effect is compensated by normalizing the amplitude of the SAW pulses in Fig. 3, the Rayleigh wave form variation is similar to that obtained in the case of a cylinder (Fig. 3 of Ref. 9).

As pointed out by Tsukahara *et al.*,[11] a collimated Rayleigh wave of nearly constant width can propagate around the equator if the length $2b=2a\theta$ of the source is appropriately chosen.

In time harmonic regime, the finite angular aperture 2θ of the line makes the Rayleigh wave to spread in a cone of angle α such that:

$$\sin\alpha = \lambda_R/b. \tag{2}$$

The curvature of the line source tends to focus the Rayleigh beam into a cone of angle β such that

$$\sin\beta = 2b/F, \tag{3}$$

where the focal length F is equal to $\pi a/2$. The two effects balance when the diverging and converging angles α and β are equal. The aperture angle θ_{col} for launching a collimated beam is expressed by

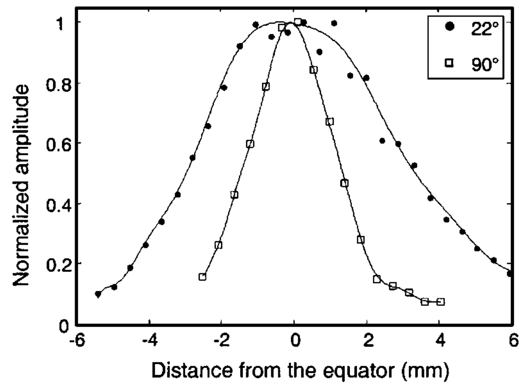

FIG. 4. Profiles of the Rayleigh wave beam measured along the meridians $\phi_1=20°$ and $\phi_2=90°$. Line source of length $2b=12$ mm on a steel sphere of radius 25 mm.

$$\theta_{col} = \sqrt{\frac{\pi\lambda_R}{4a}} \cong 1.25\sqrt{\frac{\lambda_R}{2a}} \tag{4}$$

similar to the empirical formula deduced by Ishikawa *et al.* from a numerical finite element simulation.[8]

In pulse regime, the acoustic beam can be focused only if the focal zone lies in the near field domain. According to the analysis developed by Royer *et al.* for a Gaussian thermoelastic source and a laser pulse of time duration Δ (Eq. 23 of Ref. 12), this condition expressed as

$$F = \frac{\pi}{2}a \leqslant \frac{b^2}{V_R\Delta}. \tag{5}$$

The equality corresponds to a collimated beam of constant angular aperture

$$\theta_{col} = \sqrt{\frac{\pi}{2}\frac{V_R\Delta}{a}} \cong 1.25\sqrt{\frac{V_R\Delta}{a}}. \tag{6}$$

It should be noted that, assuming a wavelength $\lambda_R=2V_R\Delta$ for the Rayleigh wave at the central frequency, Eq. (6) deduced from Eq. (4).

For our experimental conditions ($\Delta=20$ ns, $a=25$ mm, $V_R=2960$ m/s), diffraction free propagation is obtained for $\theta_{col}=3.5°$ and an optimal source length

$$2b_{col} = \sqrt{2\pi a V_R\Delta} \tag{7}$$

equal to 3.05 mm. Figure 5 shows the results of an experiment performed with a line source of length $2b=3$ mm. As

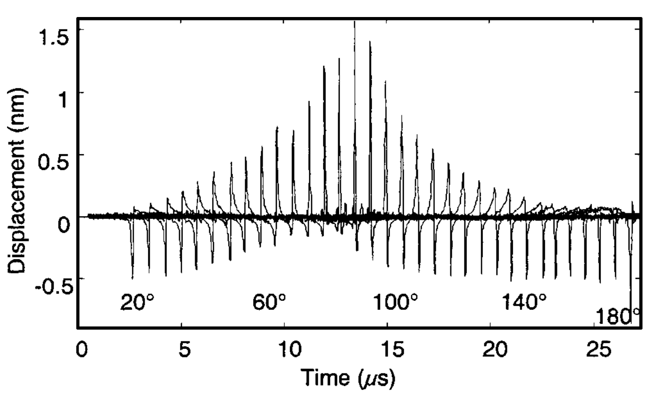

FIG. 3. Rayleigh wave forms recorded along the equator from 20° to 180° (5° step). Line source of length $2b=12$ mm on a steel sphere of radius 25 mm.

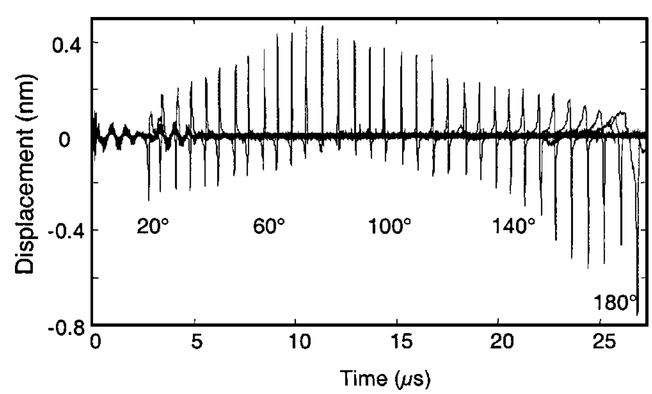

FIG. 5. Rayleigh wave forms recorded along the equator from 20° to 180° (5° step). Line source of length $2b=3$ mm on a steel sphere of radius 25 mm.

expected, the amplitude of the Rayleigh pulse is nearly constant. No focusing effect is observed at $\phi=90°$.

In conclusion, experimental and theoretical investigations of spherical surface acoustic waves generated by a thermoelastic line source show that the mechanical displacement is periodically reversed and focused. The origin of the first effect is the same that for a cylinder. The second effect due to the surface curvature is specific to the sphere. For time harmonic and pulsed regimes, geometrical conditions for launching a collimated SSAW beam have been derived and experimentally observed.

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