Nonlinear distortion of finite-amplitude ultrasonic waves investigated by optical heterodyne interferometry

Xiaoping Jia
Groupe de Physique des Solides, Universités Paris 7 et Paris 6, CNRS-URA n°17, Tour 23, 2 place Jussieu, 75251 Paris Cedex 05, France

Laszlo Adler
Industrial, Welding and Systems Engineering, The Ohio State University, 190 West 19th Avenue, Columbus, Ohio 43210-1182

Gérard Quentin
Groupe de Physique des Solides, Universités Paris 7 et Paris 6, CNRS-URA n°17, Tour 23, 2 place Jussieu, 75251 Paris Cedex 05, France

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The study of asymmetrical patterns of the light diffracted by ultrasonic waves has been utilized in the last decades to investigate ultrasonic wave distortions caused by the nonlinearity of the medium. In contrast to the investigation of diffraction patterns in space, a novel optical method is proposed in this work to analyze, in the frequency domain, the phase modulation (related to the Raman–Nath parameter) of the probe beam. Using a heterodyne interferometric method, the effects of ultrasonic waves are described as sideband generations (Doppler shift) about a central frequency (optical heterodyning) in the frequency spectrum of the photocurrent. Asymmetry resulting from nonlinear effects was observed upon the spectrum envelope. This effect increases both with ultrasonic intensity and with propagation distance. Experimental results obtained in water confirmed qualitatively the theoretical analysis which includes the second and third harmonics in the formulation. © *1995 Acoustical Society of America.*

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INTRODUCTION

Acousto-optic interaction provides a convenient way of probing ultrasonic waves in transparent media and has the advantage not to disturb the ultrasound field. Raman–Nath theory predicts that there is a symmetry of diffraction pattern about the zero order of diffraction when the light is normally incident to the plane of a sinusoidal ultrasonic wave. However, when the ultrasonic intensity increases, the ultrasonic waveform distorts due to the nonlinearity of the medium. Optical diffraction patterns have been shown to be rather sensitive to changes in ultrasonic waveforms, which reflect the distortion of ultrasonic waveform by becoming spatially asymmetrical.[1–3] This asymmetry pattern was previously observed in the optical far field, either on the diffraction intensity of individual orders by Zankel and Hiedemann[2] or on the envelope of whole diffraction patterns by Mikhailov and Shutilov,[1] and Breazeale and Hiedemann.[3] By using various inverse procedures, it is possible to deduce the ultrasonic waveform distortion from diffraction patterns.[1,4] In the present paper, we describe another optical method for investigating finite-amplitude ultrasonic distortion. This method, which combines acousto-optic interaction with heterodyne interferometry, has been utilized to measure low-intensity ultrasonic waves in fluids and in solids.[5,6] Unlike conventional optical diffraction methods, a narrow probe light is used in our experiments and the phase modulation (Raman–Nath parameter) detected in the optical near field is deduced directly owing to heterodyne interferometry. The behavior of the frequency spectrum corresponding to the signal resulted at the photodetector will be studied as function of ultrasonic distortion caused by finite-amplitude waves.

I. THEORETICAL ANALYSIS

In this section, we first review briefly the principle of the interferometric method in pressure measurements of low-intensity ultrasound, and then apply this method to detect finite-amplitude ultrasonic waves.

A. Low-intensity ultrasonic measurements

The basic principle of the heterodyne interferometry applied in pressure measurements can be readily understood by the schematic diagram illustrated in Fig. 1. A narrow light beam from a laser source (ω_L) is split into a reference part and probe part by a beam splitter (BS1). The reference beam (r), after successively reflected by M1 and BS2, is focused to a photodetector. The probe beam (s), shifted in frequency (ω_B) by a Bragg cell, normally passes through the ultrasonic beam before it mixes with the reference beam. If the acousto-optic interaction satisfies the Raman–Nath condition, the probe beam only suffers a phase modulation at the exit of the acoustic beam. The electric fields of the two beams can then be given as follows:

$$E_r=(E/\sqrt{2})\exp\, i(\omega_L t+\Phi_r),$$
$$E_s=(E/\sqrt{2})\exp\, i[(\omega_L+\omega_B)t+\nu(x,t)+\Phi_s], \quad (1)$$

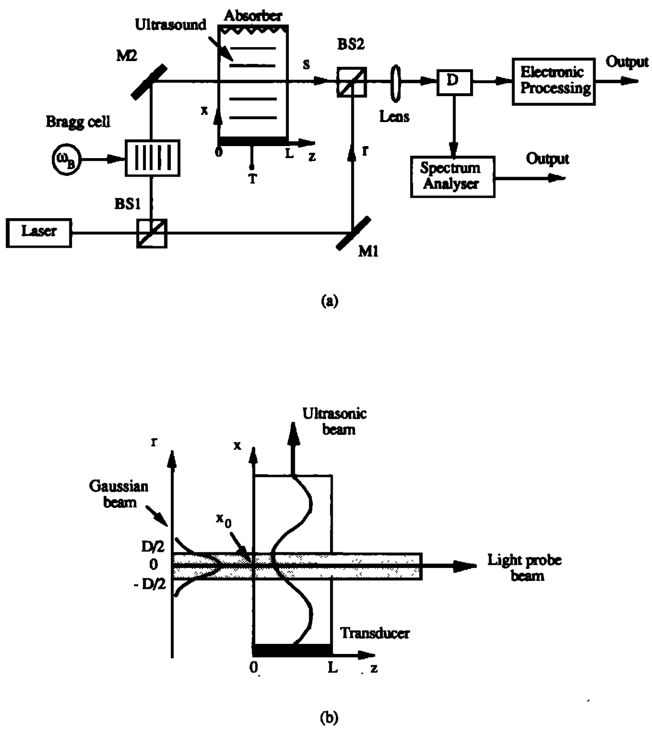

FIG. 1. (a) Diagram of the interferometric arrangement for finite-amplitude ultrasonic waves detection with M: mirror, BS: beam splitter, D: photodetector, and T: ultrasonic transducer. (b) The schema illustrating the acousto-optic interaction.

where E is the electric field amplitude of the laser source and Φ_r and Φ_s are the phase constants associated with the optical arms of the reference and probe beams, respectively. In fluid media, the Raman–Nath parameter ν is defined by

$$\nu=2\pi(\Delta n)L/\lambda_L=2\pi\kappa pL/\lambda_L, \tag{2}$$

where κ is the piezoelectric coefficient, L is the path length of light through ultrasonic field, and Δn is the maximum change of index of refraction caused by ultrasonic pressure $p(x,t)$. The interference of the reference and probe beams results in a photocurrent in the photodetector, proportional to the following intensity of the optical signal:

$$I(x,t)=|E_r+E_s|^2=I_0+(I_0/2)\cos[\omega_B t+\nu(x,t)+\Phi_0], \tag{3}$$

with $I_0=E^2$, the optical intensity of the laser source, and $\Phi_0=\Phi_s-\Phi_r$. Assume a plane sinusoidal acoustic wave propagating along the x axis, $p=p_0\sin(\omega t-kx)$. The associated parameter is given by Eq. (2), $\nu=\nu_0\sin(\omega t-kx)$ with $\nu_0=2\pi\kappa p_0L/\lambda_L$. By expressing the exponential functions in terms of the Bessel functions, the variation of the photocurrent can be rewritten as follows:

$$i(x,t)=\frac{I_0}{2}\sum_{n=-\infty}^{\infty}J_n(\nu_0)\cos[(\omega_B+n\omega)t-nkx+\Phi_0]. \tag{4}$$

The frequency spectrum of this photocurrent thus contains a central component at ω_B and symmetrical sideband components at $\omega_B\pm n\omega$, whose amplitudes are determined by Bessel functions $J_n(\nu_0)$. The ultrasonic effects can thus be described either as a phase modulation of the probe light in the time domain [Eq. (3)] or the generation of sidebands (Doppler shift) in the frequency domain [Eq. (4)]. In weak ultrasonic pressure measurement, the associated Raman–Nath parameter ν is small, so only the central component ω_B and the two sidebands $\omega_B\pm\omega$ are significant. Measuring the ratio of amplitude between the central component and its sidebands $J_1/J_0\approx\nu_0/2$ allows us to deduce quantitatively the parameter ν and therefore the ultrasonic pressure p from Eq. (2). For a width of ultrasonic wave $L=10$ mm and with a He–Ne laser, the detection sensitivity of ultrasonic pressure is about 1 mV/5 Pa in water and 1 mV/0.4 Pa in air.[5] In pulsed ultrasonic measurements, the phase modulation term $\nu(x,t)$ and then the time waveform of ultrasonic pressure $p(x,t)$ can be demodulated linearly from Eq. (3). A detection bandwidth of 10 kHz–30 MHz is available in our experiments.[5–7]

For the applications of medical diagnosis and nondestructive evaluation, several conventional optical diffraction methods have also been developed for observing the time waveforms of broadband ultrasonic pulses. These methods consist in analyzing the diffracted light distribution in either the far field (Fraunhoffer region) or the near field (Fresnel region) of the phase grating plane, induced by ultrasound. In the far-field method,[8,9] the individual Fraunhoffer diffraction orders are caused to be overlapped by placing a spatial filter in front of a photodetector and the parameter v then is determined directly from the photodetector signal. In the near field of the phase grating, the mixing of the diffracted light beams produces intensity fluctuations at the acoustic frequency and its harmonics. Cook[10] has proposed a near-field method of obtaining the parameter ν by placing a photodetector near the phase grating plane or its corresponding image after a converging lens. Riley *et al.*[11] and others[12,13] extended this method so that the value ν can be obtained with a photodetector located at the correct position where the intensity fluctuation is maximum. This result may be independent of the exact position of the photodetector placed in the Fresnel region if an optical heterodyne method is used. This heterodyne method,[14] mixing a reference beam with the nth diffraction order in the way similar to the present work, has, however, allowed only the pulse envelope to reproduce rather than the pulse waveform under study. Generally speaking, the upper frequency limit of these previous methods is governed by the diameter of the photodetector aperture which must be small compared with the ultrasonic wavelength.[10,11] A smaller aperture, however, decreases the signal-to-noise ratio, which may limit the detection sensitivity for low acoustic pressure measurements encountered in gaseous media, for example.[5]

Opposed to the conventional Raman–Nath diffraction methods, the width of the probe light beam in the present interferometric method is required to be small as compared with the acoustic wavelength, because the temporal and spatial resolutions of this interferometric method are limited by the finite size of the light beam. When the light beam becomes comparable to or greater than the ultrasonic wavelength, the signal at the output of the photodetector $s(t)$ shall be averaged by integrating $i(x,t)$ over the finite optical beams. The filtering effects of the finite light beam are particularly serious for high frequencies, as will be seen below in finite-amplitude wave measurements where high harmonics are generated.

B. Finite-amplitude ultrasonic measurements

Finite-amplitude waves are commonly referred to as ultrasonic waves having sufficiently large amplitude so that the nonlinear effects become appreciable. An initially sinusoidal ultrasonic wave distorts as it progresses through the medium. The waveform distortion increases both with the ultrasound intensity and with the distance from the acoustic source. In the frequency domain, the distortion of a sinusoidal waveform can be described as harmonics generation and the energy transfer from the fundamental to higher harmonics. Assume the state equation to be a second-order term:

$$p=P_0+A[(\rho-\rho_0)/\rho_0]+(B/2)[(\rho-\rho_0)/\rho_0]^2, \qquad (5)$$

where p and ρ are the pressure and the density, P_0 are ρ_0 are these quantities at the static state, and B/A is the nonlinear parameter of medium. The acoustic wave generated in a dissipationless fluid by a sinusoidal vibrating source located at $x=0$, $p(0,t)=p_0 \sin \omega t$, can be described by the following formula:[15,16]

$$p(x,t)=\sum_{m=1}^{\infty} p_m \sin(m\omega t-mkx)$$

$$=2p_0\sum_{m=1}^{\infty} \frac{J_m(mx/l)}{(mx/l)} \sin[m(\omega t-kx)], \qquad (6)$$

where the discontinuity distance is $l=2\rho_0(c_0)^3/[(B/A+2)\omega p_0]$. For small amounts of distortion and propagation distance ($x \ll 1$), the ratio of second harmonic to fundamental is given as

$$a_2=\frac{p_2}{p_1}\approx\frac{\pi(B/A+2)}{2\rho_0 c_0^3} fxp_0=K_2\nu_0, \qquad (7)$$

and the ratio of third harmonic to fundamental is

$$a_3=\frac{p_3}{p_1}\approx 3\left(\frac{\pi(B/A+2)}{2\rho_0 c_0^3} fxp_0\right)^2=K_3\nu_0^2. \qquad (8)$$

Here K_2 and K_3 are the coefficients defined by Eqs. (2), (7), and (8), which measure the magnitude of ultrasonic distortion caused by harmonics generation as functions of distance x and frequency f.[2,17] In the case of a 1-MHz and 20-mm-wide ultrasonic wave traveling in water, for instance, K_2 and K_3 are evaluated as 2.3×10^{-3} and 1.6×10^{-5} for $x=2$ cm, respectively, and 2.6×10^{-2} and 2.0×10^{-3} for $x=22$ cm.

To investigate the propagation of finite-amplitude waves with the interferometric method, we shall determine the induced phase modulation of the probe light. Substituting Eq. (6) into Eq. (2) leads to

$$\nu(x,t)=\sum_{m=1}^{\infty} \nu_m \sin[m(\omega t-kx)], \qquad (9)$$

where $\nu_m=2\pi\kappa p_m L/\lambda_L$ and p_m is the amplitude of mth harmonics. Rewriting Eq. (3) with Eq. (9) and expressing the exponential function in terms of the Bessel functions, the photocurrent i can be expressed as follows:

$$i(x,t)=\sum_{n=-\infty}^{\infty} \Psi_n \cos[(\omega_B+n\omega)t-nkx+\Phi_0], \qquad (10)$$

where

$$\Psi_n=\frac{I_0}{2}\sum_{k_2=-\infty}^{\infty}\cdots\sum_{k_n=-\infty}^{\infty} J_{k_1}(\nu_1)\cdots J_{k_n}(\nu_n)\cdots, \qquad (11)$$

with $k_1=n-2k_2-3k_3-\cdots-nk_n-\cdots$. The expression of Eq. (11) was first obtained by Zankel and Hiedemann in the finite-amplitude wave investigation,[2] and by Leroy[18] and Heignbors and Mayer[19] in weak pulsed wave study. In contrast to the previous work, ω_B (=70 MHz) illustrated here in Eq. (10) is the Bragg cell driving frequency instead of the optical frequency ω_L ($\sim10^{14}$ Hz). As a matter of fact, optical diffraction methods used previously in finite-amplitude wave measurements generally consisted of measuring, in the optical far field, the spatial distributions $|\Psi_n|^2$ of the diffracted orders averaged in time. Now using optical heterodyning enables us to detect $|\Psi_n|$, in the optical near field, as the amplitudes of acoustically generated sidebands ($\omega_B \pm n\omega$) in the frequency domain with a conventional spectral analyzer. In addition, the interferometric method needs one photodetector for only one measurement, so the ratio of signal-to-noise of the whole frequency spectrum will be improved (see

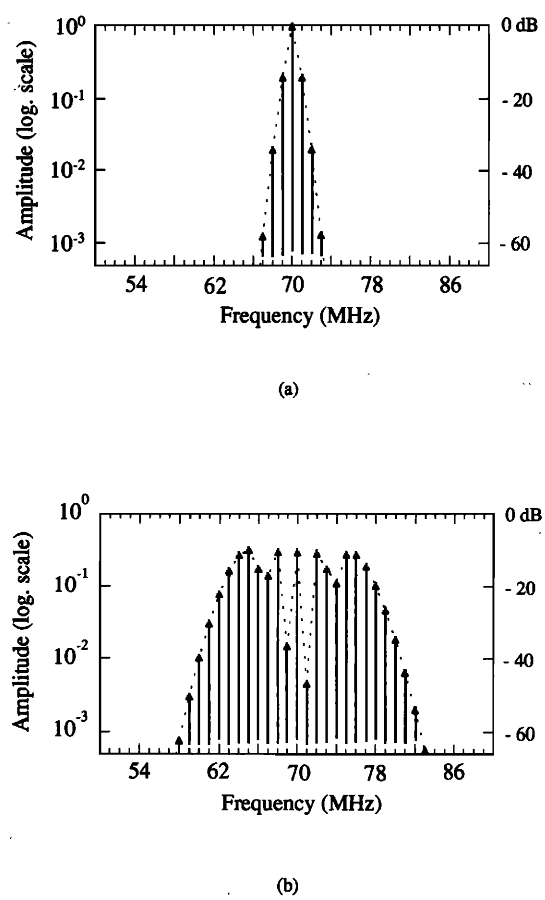

FIG. 2. Frequency spectra calculated with $f=1$ MHz and $K_2=2.3\times10^{-3}$, $K_3=1.6\times10^{-5}$, for $\nu_0=0.4$ (a) and 7 (b), respectively.

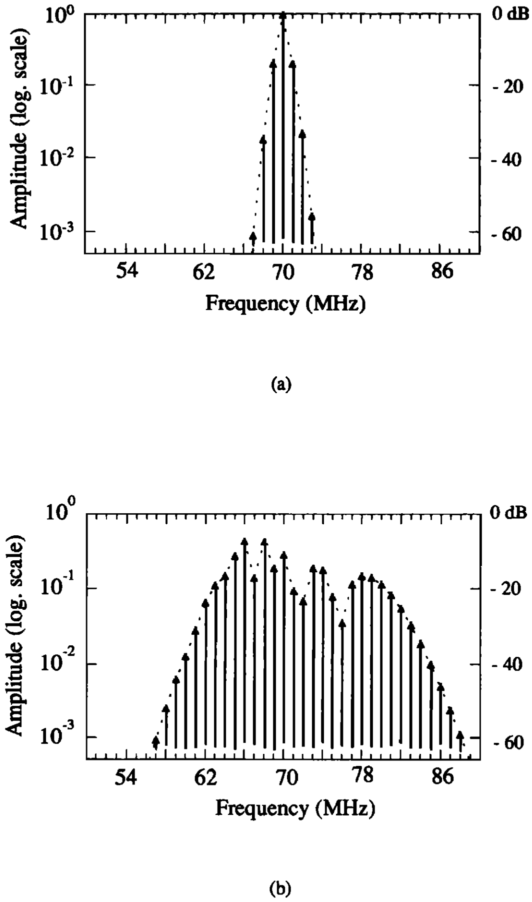

FIG. 3. Frequency spectra calculated with $f=1$ MHz and $K_2=2.6\times10^{-2}$, $K_3=2.0\times10^{-3}$, for $\nu_0=0.4$ (a) and 7 (b), respectively.

the results in Sec. II) in comparison with conventional optical diffraction patterns, which are often obtained by more than one photodetector or several measurements. For small amounts of distortion, if only the fundamental, second, and third harmonics are considered in Eq. (11), the amplitude of the sideband $\omega_B+n\omega$ turns out to be, using Eqs. (7) and (8),

$$\Psi_n=\frac{I_0}{2}\sum_{k_2=-\infty}^{\infty}\sum_{k_3=-\infty}^{\infty}J_{n-2k_2-3k_3}(\nu_0)\times J_{k_2}(K_2\nu_0^2)J_{k_3}(K_3\nu_0^3). \quad (12)$$

As mentioned previously, the filtering effects of the finite laser beam are important in nonlinear acoustic measurements. Assume that a Gaussian light beam, $E(r)=E_0\exp[-4(r/D)^2]$ of diameter $D\approx0.1$ mm, intersects a plane ultrasonic wave at a distance x_0 from the transducer. Focusing by a converging lens onto the photodetector, the interference of the light beams gives rise to a signal $s(x_0,t)$ equal to $i(x,t)=[E^2(r)/2]\cos[\omega_Bt+\nu(x,t)+\Phi_0]$ integrated over the whole beam. As shown in Fig. 1, using coordinate transformation $x=x_0+r$ and the expression developed in Eq. (10), we can obtain the frequency spectrum of the signal at the output, as follows:

$$\begin{aligned}s(x_0,t)&=\int_{-\infty}^{\infty}i(x,t)dr\\&=\mathrm{Re}\Bigg\{\sum_{n=-\infty}^{\infty}\Psi_n\exp i[(\omega_B+n\omega)t+\Phi_0]\\&\quad\times\int_{-\infty}^{\infty}\exp\left[-8\left(\frac{r}{D}\right)^2\right]\\&\quad\times\exp i[nk(x_0+r)]dr\Bigg\}\\&\propto\sum_{n=-\infty}^{\infty}\Psi_n\exp\left(-\frac{(nkD)^2}{32}\right)\cos[(\omega_B+n\omega)t\\&\quad-nkx_0+\Phi_0].\end{aligned} \quad (13)$$

When the optical beamwidth D is small enough compared with the acoustic wavelength, the correction term $\exp[-(nkD)^2/32]$ due to the filtering effect is negligible. Equation (13) is then reduced to Eq. (11) but with $x=x_0$ as would be expected. It is noted that the higher the ultrasonic frequency is, the more important the optical filtering effects are. Equation (14) is obtained by neglecting the photodetector aperture (R). It is valid as long as $D<R$. In conventional optical methods, a narrow light beam often is not desirable

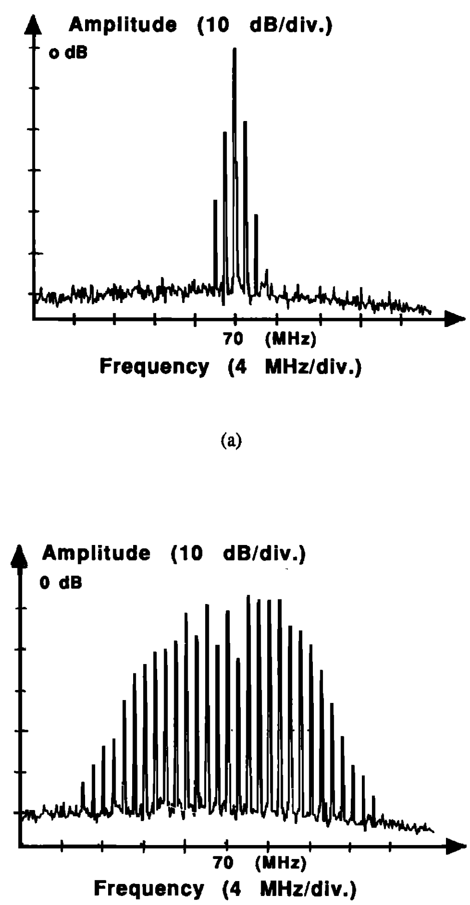

FIG. 4. Experimental frequency spectra detected at $x=2$ cm from 1-MHz transducer driven for ultrasonic intensity close to (a) $\nu_0=0.4$ and (b) $\nu_0=7$, respectively.

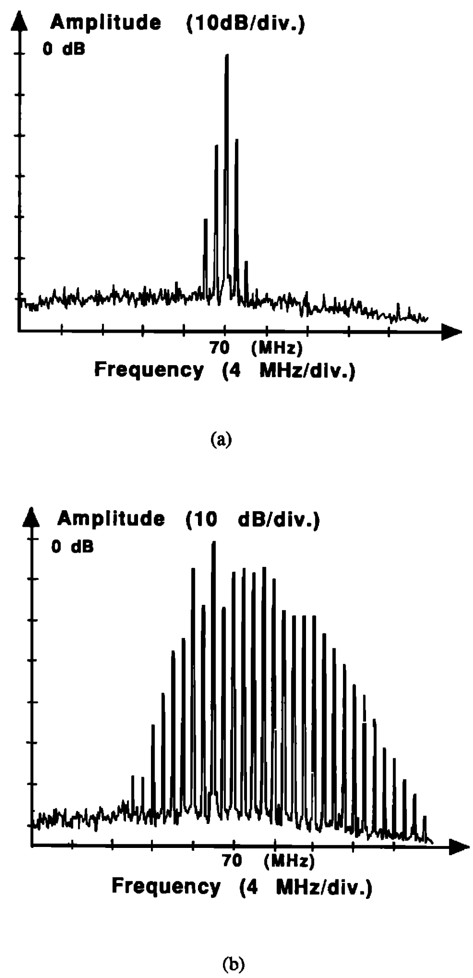

FIG. 5. Experimental frequency spectra of ultrasonic waves generated by the same transducer in Fig. 5, but detected at $x=22$ cm.

for obtaining the discrete diffraction patterns. For example, Breazeale[3] and Hargrove *et al.*[20] have published some striking photographs that illustrate the far-field diffraction patterns produced by both wide and narrow light beams. A narrow light beam caused an overlapping of different orders and the image appeared broadened.

Figure 2 illustrates the spectra of $s(x_0,t)$ plotted in logarithm scale and calculated with Eq. (13) for $\nu_0=0.4$ and 7, respectively. The values of K_2 and K_3 were taken as 2.3×10^{-3} and 1.6×10^{-5}, corresponding to $f=1$ MHz and $x=2$ cm. Figure 3 presents similar spectra calculated with greater values of $K_2=2.6\times10^{-2}$ and $K_3=2.0\times10^{-3}$ for $f=1$ MHz and $x=22$ cm. It is seen that at weak ultrasonic intensity ($\nu_0\leq0.4$), few sidebands are present in the frequency spectrum [Figs. 2(a) and 3(a)]. Whatever the magnitude of nonlinear effects characterized by K_2 and K_3 is, the resulting frequency spectra remain symmetrical about the central component as predicted by the Raman–Nath theory. When ultrasonic intensity becomes higher, a large number of sidebands are produced in the frequency spectrum [Figs. 2(b) and 3(b)]. Moreover, the envelope of the frequency spectrum turns to be asymmetrical both with increasing ultrasonic intensity and propagation distance from the source. Notice that the larger the ultrasonic intensity and nonlinear effects ($\propto K_2,K_3$) are,

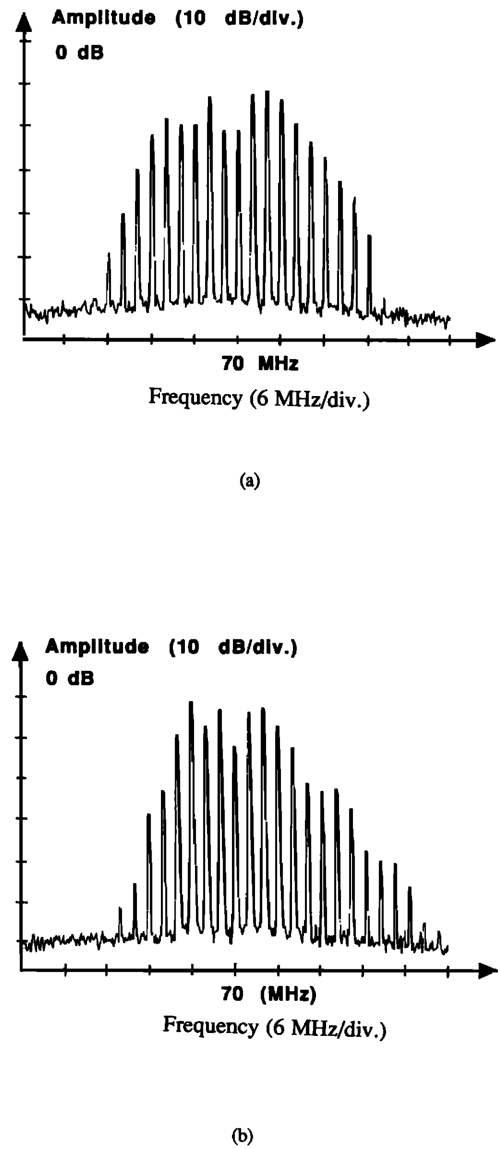

FIG. 6. Experimental frequency spectra of 2-MHz transducer excited for ultrasonic intensity $\nu_0\approx7$, detected (a) at $x=2$ cm and (b) at $x=22$ cm.

the more remarkable is the asymmetry, leaving more sidebands generated at the right of the central component. Such behavior of the frequency spectrum is very similar to the evolution of spatial diffraction patterns.[3]

II. EXPERIMENTAL RESULTS

Ultrasonic waves were generated in water by narrowband PZT and quartz transducers. High ultrasonic intensities were attained by using a power amplifier. For a transducer working at a given frequency, ultrasonic pressures were detected by a heterodyne interferometer as a function of the Raman–Nath parameter ν_0 for various distances from the source. Details of the experimental apparatus can be found elsewhere.[5,6] The proportionality between the driving electric voltage and the parameter ν_0 was determined at weak pressure measurement. Typical spectra of ultrasonic pressures excited by a 1-MHz and 20-mm-diam transducer are shown in Fig. 4. They were detected at $x=2$ cm from the transducer driven to values of ν_0 of about 0.4 and 7, respectively. As predicted by theoretical calculations (Fig. 2), the envelope of the frequency spectra remain almost symmetrical, keeping the same number of sidebands around the central component $f_B=70$ MHz. Figure 5 shows the spectra of ultrasonic pressures generated by the same source but detected at a greater distance $x=22$ cm. As clearly seen in Fig. 5(b), a significant asymmetry is induced in the frequency spectrum. The fact that more sidebands were generated at right than at left of the central component agrees qualitatively with the theoretical prediction [Fig. 3(b)]. Measurements performed with a 2-MHz transducer for $\nu_0 \approx 7$ are also presented in Fig. 6, demonstrating again that the asymmetry of the spectrum increases with distance. Besides, it is noted that a much larger number of sidebands has been detected in this work compared with that obtained by the optical diffraction method with finite but moderate acoustic amplitude.[3] As mentioned in the previous section, this is due to the good ratio of signal-to-noise provided by the present interferometric method. This advantage makes it possible to investigate the envelope evolution of the frequency spectrum even with a moderate acoustic amplitude.

III. SUMMARY

Finite-amplitude ultrasonic waves propagating in water have been observed by an optical heterodyne interferometric method. Asymmetrical patterns in the frequency spectra of the photocurrents have been detected when the ultrasonic distortions became important. A qualitative agreement was obtained between the experimental results and the theoretical predictions. Compared with the conventional optical diffraction methods, the interferometric method offers a better signal-to-noise ratio in the frequency spectrum, which enables one to study the envelope behavior of the frequency spectra as a function of ultrasonic distortion with finite but moderate acoustic amplitude. In future, we will try to improve the optical alignment in order to measure quantitatively intensity variations of individual sidebands as a function of wave distortion.

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